HW1 Solutions

Problem 1 a) Algorithm: Color each edge either red or blue with equal probability. Let X_i be the event that i^{th} triangle is monochromatic.Then $Prob[X_i] = 2(\frac{1}{2^3}) = 1/4$. Expected number of monochromatic triangles are $\binom{n}{3}/4$. b) Order the edges $(e_1, \ldots, e_{\binom{n}{2}})$. Suppose G_i be the partially colored graph after coloring first i edges. For each choices of the color of the edge e_{i+1} compute the expected number of triangles that are monochromatic in G_i. Choose the color which maximizes the expected value. Example, if a triangle T is colored $rr*$ then $Prob[T \text{ is mono}|x_{i+1} = r, \ldots] = 1$ and $Prob[T \text{ is mono}|x_{i+1} = b, \ldots] = 0$.

Problem 2 a) Sort in ascending order of ranks. Choosing the best option (lowest rank) currently available and remove all companies that clashes with the current choice. Use the exchange argument to prove optimality.

b) This is a transversal matroid. Ground set is C. Let $\mathcal{H}$ be the collection of H_i's. A subset $A \subseteq C$ is independent if there a subset $B \subseteq \mathcal{H}$ with $|A| = |B|$ such that A is system of distinct representative of B (also known as a transversal). We can represent $(C, \mathcal{H})$ as a bipartite graph G, where (c, H) is an edge if $c \in H$. Then a transversal corresponds to a matching in G. Given two such matchings M and M', with $M < M'$, we can augment M (see matching notes). Hence the independent sets satisfy the augmentation property. They clearly satisfy the hereditary property. A basis of $(C, \mathcal{H})$ corresponds to maximum matching. The weighted version thus gives an optimal schedule that maximizes the number of interviews.

Problem 3 Replace the edge with capacity r with a unit capacity edge. Choose a $(1 - r)$ unit flow from s to t along the new edge which does not go through any of the other unit capacity edges.