Homeowrk 2

Due On: October 14, 2020 2:59PM (CDT)

Problem 1(3 pts) Let A be an $n \times n$ matrix with each entry equal to either 0 or 1. We can apply two types of operations on A. A row operation $R(i,j)$ swaps the row i with row j and vice versa. Similarly, we define a column operation $C(i,j)$ on the pair of columns i and j. Given A, determine in polynomial time if there is a sequence of row or column operations such that the resulting matrix B has the following property: $\textsc{trace}(B - I) = 0$.

Problem 2. (4 pts) Suppose $G(V, E)$ is a graph of all S&T students as follows. Vertices (V) of G represents students and two students share an edge if they know each other. A triangle is a group of three students who all know each other. We define a property of G, called triangle-density, as a measure of social connections within a group of students. For a subset $A \subseteq V$ of students let $tr(A)$ be the number of triangles in the graph $G[A]$ induced by vertices in A. Then the triangle-density is the average number of triangles a student is part of ($tr(A)/|A|$). For some parameter λ (a rational) determine in polynomial time if there is a group of students whose triangle-density is at least λ.

Problem 3. (3 pts) Suppose that at some point in the execution of a push-relabel algorithm, there exists an integer $0 < k < n$ for which no vertex u has $h(u) = k$. Show that all vertices with $h(u) > k$ are on the source side of a minimum cut. If such a k exists, the gap heuristic updates every vertex $u \in V \setminus \{s\}$ for which $h(u) > k$, to $h(u) = \max(h(u), n+1)$. Show that this operation does not violate the height constraints. Explain why this may be useful in practice.