Homework-1

Due On: September 28, 2020 2:59PM (CDT)

Problem 1. (4pts) Suppose we color the edges of a complete graph with n-vertices (K_n) either red or blue. We say a triangle is monochromatic if all its edges are of the same color.

1. Give a randomized algorithm that produces a bi-coloring of the edges such that the number of monochromatic triangles is $\frac{\binom{n}{3}}{4}$ in expectation.

2. Using the method of conditional expectation develop a polynomial time deterministic algorithm the produces a coloring with at most $\frac{\binom{n}{3}}{4}$ monochromatic triangles.

Problem 2. (4pts) You have been selected for multiple onsite job interviews from the set of companies $C = \{c_1, \ldots, c_n\}$ in the coming week. You have ranked these companies according to your preference (lower is better). However, you cannot go to all the interviews due to time conflicts and can only attend one interview each day (they are located in different cities). Even after speaking with the respective HR's for several hours you did not make any progress. Let $H_i \subseteq C$ be the set of companies with whom you can schedule an interview on the ith day ($i \in \{M, T, W, R, F\}$).

1. Give a greedy algorithm that schedules an interview on each day (if such an schedule exists) such that the total rank of the selected companies is minimized. Prove it correctness.

2. Determine the matroid hidden in this problem. You do not have to formally prove it.

Problem 3. (2pts) Show that even if all the capacities are integral but the initial flow value is irrational then the generic algorithm (Ford-Fulkerson) may not terminate or find the maximum flow. [Hint: Use the example given in class and modify the capacities accordingly.]