CS 5200 Homework - 5

Instructor Avah Banerjee

May 1, 2024, 10 PM CST

Problem 1 (30 pts) Given a positive edge-weighted graph $G = (V, E, w)$ and a source vertex s, design an algorithm that can determine the number of shortest path trees with respect to s.

Problem 2 (30 pts) (Modular Shortest Paths) Suppose we define the length of a path $(s = u_0, u_1, \ldots, u_{k-1} = u)$ in a positive integer-weighted graph $G = (V, E, w)$ as:

$$dist_p(s, u) = \left(\sum_j w(u_j, u_{j+1}) \right) \mod p$$

for some fixed integer $p > 0$. Can we still use Dijkstra's algorithm to compute modular shortest paths? Justify your answer.

Problem 3 (40 pts) Suppose you are given a graph $G = (V, E, w)$ with positive integer edge weights and the following twist: for a subset of edges $M \subset E$, the weights are unknown. Along with G, you are also given a shortest path tree T_s with respect to some source vertex s. Propose an algorithm that can determine, given the input (G, M, s, T_s), whether there is a valid assignment of weights to the edges with missing weight information such that T_s remains a valid shortest path tree with respect to the new weight assignment. Also determine the running time of your algorithm. Justify your answers.