# CS 5200 PSET -2

Avah Banerjee

Due on: February 14, 11:59PM

**Problem 1** We say a list $L = (x_1, \ldots, x_n)$ is $k$-sorted (for some $k \in [n]$) if $\forall i, j \in [n] : i+k \leq j \implies x_i \leq x_j$. An 1-sorted list is simply a sorted list. An example of a 3-sorted is : $A = (1, 2, 4, 3, 6, 7, 5)$. However this list is not 2-sorted since $A[5] > A[7]$. Given an unoreded list design a deterministic algorithm that can $k$-sort the list with $O(n \log \frac{n}{k})$ comparisons. Is this bound tight?

**Problem 2** Is there a deterministic algorithm that can find the $2^{nd}$ smallest element in an unsorted array with $n + \log n + O(1)$ comparisons?

**Problem 3** You are given two unordered lists $A$ and $B$ of equal length $(n)$ and containing the same set of keys. For example $A = \{2, 3, 1, 5\}$ and $B = \{1, 5, 3, 2\}$. Your task is to design a randomized algorithm that can sort both $A$ and $B$ with $O(n \log n)$ comparisons in expectation. However, you can only compare two keys if they come from different lists.

**Problem 4** Let $X$ be a set of $n$ keys and let $S$ be a set of $m$ subsets of $X$. Design a deterministic algorithm that can find the maximum of every subset in $S$ with $O(n \log n)$ comparisons.