CS 5200 Homework - 5

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Due: December 1, 11:59PM

Problem 1

Given a set X containing n distinct points in a two-dimensional plane, each point represented as $p_i(x_i, y_i)$, your task is to develop a dynamic programming algorithm. The algorithm should identify the longest chain of points in X where each point in the chain strictly dominates the previous one. A point $p(a, b)$ is said to dominate another point $q(c, d)$ if and only if $a \geq c$ and $b \geq d$ and at least $a \neq c$ or $b \neq d$.

Example
Input:
$n = 5$
Points: $(1, 4), (2, 3), (3, 5), (5, 1), (2, 2)$
Output:
Length of the longest chain: 3
Chain: $(3, 5) \rightarrow (2, 3) \rightarrow (2, 2)$

Task 1: Construct the dynamic programming graph as well as the dynamic programming recurrence relation. Determine if your solution satisfies the optimal substructure property. Finally, using the dynamic programming graph, determine the running time of your algorithm.

Task 2: Implement your algorithm in Python. The input-output format of your algorithm is as shown above. It takes a list of tuples and returns both the length and the longest chain.

Problem 2

You are given a rooted tree $T = (V, E)$ with n nodes and the root r. Each node $u \in V$ has an integer label $l(u)$. Suppose $S \subseteq V$. We say S is well-formed if for every $u, v \in S$ if u is an ancestor or the parent of v then $l(u) \leq l(v)$. Let $wf(T)$ be the maximum size of any well-formed set in T. Give a dynamic program to compute $wf(T)$.

1. Determine the dynamic program DAG.
2. Determine the dynamic program recurrence.
3. Determine the running time and space usage.

4. Can you also find a set S such that $|S| = wf(T)$?

Extra Credit. Extend your solution to the case when, instead of a tree, you now have a DAG. The definition of well-formedness still follows from above, where we say u is an ancestor of v if there is a directed path from u to v.