CS 5200 Homework - 3

Instructor Avah Banerjee
Due: October 31, 11:59PM

Problem 1 Suppose $X = \{x_1, \ldots, x_n\}$ is a list of n elements with an unknown ordering π (we may assume that n is a power of 2). When selecting a pivot p from X we choose it uniformly at random from X. Then what is the probability that two given elements, x_i and x_j, will be located in different partitions of the list according to the pivot p?

Problem 2 Suppose $X = \{x_1, \ldots, x_n\}$ is a list of n-elements with an unknown ordering. Consider the following randomized sorting algorithm.

Algorithm 1 Random Pair Sorting Algorithm

```
1: procedure RandomPairSort(X)
2:     Input: X = {x_1, ..., x_n}
3:     while X is not sorted do
4:         Pick a random pair (x_i, x_j) such that 1 ≤ i < j ≤ n
5:         if i < j and x_i > x_j then
6:             Swap x_i and x_j
7:         end if
8:     end while
9: end procedure
```

Determine on average how many comparisons the above algorithm makes.

Problem 3 Every tree $T = (V, E)$ (an undirected, connected, simple graph with $n - 1$ edges) has at least one vertex x such that removing x (as well as all edges incident to it) disconnects the tree into two or more subtrees, each with at most $\frac{n}{2}$ vertices. Given a tree T, devise an algorithm that identifies x and outputs all the disjoint subtrees resulting from the removal of x. Implement this algorithm in Python (using a Jupyter notebook). Test your algorithm with trees of sizes $n = 100, 500, 1000, 5000, 10000$. For each value of n, compute the average running time using 30 input instances. For this test, you must generate trees randomly. Employ the following recursive procedure: suppose you have generated a tree with $n-1$ vertices. To add the n^{th} vertex, select a vertex from the existing tree and connect it to the new vertex.