CS 5200 Homework - 1

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August 31, 2023

Problem 1 Given the following list of functions:

1. $f(n) = 2n^{1/n}$

2. $g(n) = n^2 \log n$

3. $h(n) = n^{\log n}$

4. $i(n) = 2^{2^{\log n}}$

5. $j(n) = (\sum_{i=0}^{n} 2^i)^{\frac{1}{\log n}}$

Arrange them in increasing order of growth rate as n approaches infinity.

Problem 2 Consider two sequences, $\{a_n\}$ and $\{b_n\}$, defined as:

$$a_0 = 2,$$
$$a_1 = 3,$$
$$b_0 = 1,$$
$$b_1 = 4,$$

For $n \geq 2$, the sequences are defined as:

$$a_n = 3a_{n-1} - b_{n-1}$$
$$b_n = -a_{n-1} + 2b_{n-1}$$

1. Represent the above recurrence relations as matrix multiplications. Specifically, aim to express:

$$\begin{bmatrix} a_n \\ a_{n-1} \\ b_n \\ b_{n-1} \end{bmatrix} = M \times \begin{bmatrix} a_{n-1} \\ a_{n-2} \\ b_{n-1} \\ b_{n-2} \end{bmatrix}$$

Where M is a 4x4 matrix. Determine the entries of M.

2. Using the matrix M derived in the previous step, express a relationship to compute

$$\begin{bmatrix} a_n \\ a_{n-1} \\ b_n \\ b_{n-1} \end{bmatrix}$$

in terms of the base values a_1, a_0, b_1, and b_0.

Problem 3   Given two positive integers x and y, write an algorithm that computes their lowest common multiple. Prove its correctness and determine the number of atomic steps it takes in the worst case. You may assume that operations involving a constant number of bits take constant time and, thus, can be thought of as an atomic step.