CS 5200 Final Exam

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Instructions You should submit your solution within a single PDF document. The PDF should be typed in (either in Word or LaTeX) and not scanned handwritten work.

Problem 1
Input: An array $A = [a_1, a_2, \ldots, a_n]$ of n distinct integers, and a permutation pattern $\pi = [\pi_1, \pi_2, \ldots, \pi_k]$ of length k. **Goal:** Find the longest subsequence S of A such that the relative order of elements in S matches π. More formally, for $S = [a_{i_1}, a_{i_2}, \ldots, a_{i_m}]$, we require that for all $1 \leq u, v \leq m$, $\pi_{1+(u \mod k)} < \pi_{1+(v \mod k)}$ if $a_{i_u} < a_{i_v}$.
Questions:

1. Give an efficient algorithm (possibly dynamic programming based) to solve the problem for arbitrary k (time complexity both depends on n and k).
2. If k is constant, can this problem be solved in linear time? If so, find a linear time algorithm for this problem.

Problem 2
Input: A connected, weighted undirected graph $G = (V, E, w)$, and two distinct minimum spanning trees T_1 and T_2. **Goal:** Find a sequence of edge swap operations that transforms $T_1 = (V, E_1)$ into $T_2 = (V, E_2)$, where:

- Each intermediate structure is a valid spanning tree.
- The number of edge replacements is minimized (should in fact be equal to $|E_1 \setminus E_2|$ or $|E_2 \setminus E_1|$).
- Among all such sequences, the maximum weight of any intermediate spanning tree is minimized.

Questions:

1. Develop a greedy algorithm and determine its approximation ratio.
2. If G is a planar graph, come up with an $O(n^2)$ (or better) exact algorithm for this problem.

Problem 3:
Input: A directed graph $G = (V, E, c, b)$ with edge capacities $c : E \to \mathbb{R}_{\geq 0}$, a designated source node $s \in V$, and two sink nodes $t_1, t_2 \in V$. **Goal:** Compute a feasible flow f from s such that:

1. The flow is conserved at all non-terminal nodes:

$$\sum_{(u,v)\in E} f(u,v) - \sum_{(v,u)\in E} f(v,u) = 0 \quad \forall v \in V \setminus \{s, t_1, t_2\}.$$

2. The capacity constraints are satisfied:

$$0 \leq f(u,v) \leq c(u,v) \quad \forall (u,v) \in E.$$

3. The total flow from s to t_1 and t_2 is maximized:

$$\text{maximize } F = f(s \to t_1) + f(s \to t_2).$$

4. Subject to the additional balancing constraint (b):

$$|f(s \to t_1) - f(s \to t_2)| \leq b.$$

Design an algorithm to compute such a flow (if exists) that maximizes the total flow from s to t_1 and t_2, while keeping the distribution of flow as balanced as possible between the two sinks.
Questions:

- Can you construct a linear program for this problem?
- Either extend the augmenting path based algorithm or come up with your own algorithm to compute the max flow and determine its running time.