CS 5001 Homework - 2

Instructor Avah Banerjee

Due: March 22, 2024 (12 Noon CST)

Problem 1 (25 pts) Let $\psi = U|00\rangle$ where $U = (Y_1 \otimes H_2)(\Lambda_1(H_2))(\Lambda_1(X_2))$. Measure the observables $\Lambda_1(Z_2)$ and $Z_1 \otimes X_2$ on the state ψ. For each observable, determine the eigenvalues (measurement outcomes) of the observable, the states after observation of a particular outcome, the probability of each outcome, and the expected value of each observable.

Problem 2 (35 pts) Consider the following 2-qubit circuit $C(U) = (H_1 \otimes I_2)(S_1^\dagger \otimes I_2)(\Lambda_1(U_2))(H_1 \otimes I_2)$, where U is some arbitrary single-qubit unitary. Suppose we apply $C(Y)$ to the state $\phi = |0\rangle \otimes (\alpha|0\rangle + \beta|1\rangle)$ (where $\alpha, \beta \in \mathbb{C}$ and $|\alpha|^2 + |\beta|^2 = 1$), then measure the observable $Z_1 \otimes I_2$ (that is, measure the first qubit). Then show that the expected value of the measurement is $\alpha\beta^* - \alpha^*\beta$.

Problem 3 (40 pts) Implement Simon's algorithm for a specific function f. For this, we will use a specific function f and, using its specification, you will create the circuit U_f used in Simon's algorithm (note that f is not a black-box function anymore as we know its implementation). $f : \{0,1\}^n \to \{0,1\}^n$ such that $f(x) = f(x \oplus s) = \overline{y}$, where $y = \min(x, x \oplus s)$, and min is taken over the values of the binary string when converted to integers and $\overline{y}$ is the compliment of y. For your numerical simulation you can take n to be 5 and choose s to be 10101. You might want to consider taking $n = 3$ and $s = 101$ if you are having difficulty creating the reversible circuit. There are a few reversible circuit generators (from Boolean expressions), such as RevKit, but they may be difficult to use due to their age.