

CS 5001 Homework - 1

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Due: Feb 14, 2024 (12 Noon CST)

Problem 1 (25 pts) The `peres` gate is defined as: $f_{\text{peres}} : \{0,1\}^3 \rightarrow \{0,1\}^3$ and $(x, y, z) \rightarrow (y \oplus z, y, x \oplus yz)$, where yz is shorthand for y **and** z . Create the `peres` gate using only `toffoli` gates and possibly additional ancilla bits. Additionally, using `peres` gate(s), create `and`, `or`, `nand`, `nor` gates. [**Bonus (10 pts)** - reset the garbage in your construction of the `peres` gate using `toffoli` gates.]

Problem 2 (15 pts) Show that it is not possible to construct the `toffoli` gate using only `cnot` gates.

Problem 3 (20 pts) The majority function ($f_{\text{maj}} : \{0,1\}^3 \rightarrow \{0,1\}$) on three bits is defined as $(x, y, z) \rightarrow xy \vee yz \vee zx$. Implement this function reversibly using gates from the following set `{toffoli, cnot}` possibly using ancilla bits.

Problem 4 (40 pts) Write a program in Python that takes as input a positive integer n and returns a circuit $C_{\text{adder}}(n)$ which can reversibly compute the sum of two n -bit numbers. You need to use the <https://www.ibm.com/quantum/qiskit> module, which allows you to use basic reversible gate operations as well as graphically output the circuit generated by your program.