Final Project

Due On: May 7, 2021 11:59PM (CDT)

1 Find the minimum of an unordered list

In this project you are tasked with implementing the quantum minimum finding algorithm by Durr and Høyer as described in their paper [1]. The algorithm is sketched below:

1. We start by picking a index $y \in [N-1]$ (where $[N-1] = \{0, \ldots, N-1\}$) from the list L ($|L| = N$) (u.a.r) and initialize a state $|\psi\rangle_0$ which is the uniform superposition of $\{|n\rangle \mid n \in [N-1]\}$. That is $|\psi\rangle_0 = \frac{1}{\sqrt{N}} \sum_n |n\rangle$. This register can be represented by $\lceil \log N \rceil$ qbits.

2. Next we create a Grover's oracle U_y using the selected element $L[y]$. Here U_y is given by, $U_y |n\rangle = |n\rangle$ if $L[n] \geq L[y]$ and $U_y |n\rangle = -|n\rangle$ if $L[n] < L[y]$. We use this oracle U_y in the Grover's search subroutine. To implement U_y you have to create a (reversible) classical circuit that compares two $\lceil \log N \rceil$-bit numbers.

3. Now we call the Grover's search subroutine to amplify the the marked states. (See Grover's algorithm with unknown number of solutions for details) Suppose y' is the outcome after measuring the register. If $L[y'] < L[y]$ we set $y = y'$ for the next round.

4. We repeat steps 1 through 3 until the total number of calls to the Grover oracle (over all iterations) exceeds m. The parameter m is discussed in the next section.

2 Things you have to do

Here is the list of things you need to do in your experiment.

1. Implement the above algorithm in Qiskit. This should be a standalone python implementation which uses the Qiskit library. Do not implement this using the web based circuit composer.

2. Take N to be 16. Generate the list L for different values of N randomly as follows: L is a random permutation of the numbers 0 through 15. Using the solution from HW3 pick a random permutation π and use it to shuffle the elements in L.

3. Plot the histograms of the probabilities of the basis states in $|\psi\rangle_2$, the state after invoking the Grover's search routine. Do this for each round. Verify that the (probability) amplitude of the marked states are getting amplified.

4. Since we are using smaller values of N the expression given in the paper may be too loose for us. So you have to find the appropriate m by experimenting with different values. (I will discuss this a bit more in class)

3 What you should submit

1. Your Python Code (Make sure to test it using the latest version of Python and Qiskit)
2. A report (I will send you a template later), in which you briefly describe the problem and you implementation (to not copy paste code) and present your results.
3. I will also separately interview each group and ask you questions about the project.

References and Further Reading

[1] Christoph Durr and Peter Hoyer. A quantum algorithm for finding the minimum. *arXiv preprint quant-ph/9607014*, 1996.