Homework 2

Due On: March 4, 2021 3:30PM (CST)

Problem 1 (Reversible Circuits) Construct the Toffoli gate using ony the Fredkin gate. Make your construction reversible. Further, any ancilla (extra) bits used must be returned to 0 after computation using the uncomputation trick.

Problem 2 (Amplification) Let $|\eta\rangle$ and $|\zeta\rangle$ be two nearly orthogonal states where $|\langle\eta,\zeta\rangle| = \epsilon \in (0,1)$. Now consider the two transforms :

$$U_\eta = I - 2|\eta\rangle\langle\eta| \quad \text{and}$$
$$U_\zeta = I - 2|\zeta\rangle\langle\zeta|$$

Let,

$$|\zeta(k)\rangle = \underbrace{U_\eta U_\zeta \cdots U_\eta U_\zeta}_{k-times} U_\eta U_\zeta |\eta\rangle$$

Let $\cos\theta_k = \langle\eta,\zeta(k)\rangle$. Write a python program, that given $|\eta\rangle, |\zeta\rangle$ plots θ_k against k. Experimentally verify if θ_k ever reaches 0 and if so for what minimum value of k. Can you find any relation between this value and ϵ.

Problem 3 Let,

$$A(\alpha,\beta,\gamma) = \begin{bmatrix} \alpha & \beta - i\gamma \\ \beta + i\gamma & -\alpha \end{bmatrix} \tag{1}$$

where $\alpha,\beta,\gamma \in \mathbb{R}$. Show that $A(\alpha,\beta,\gamma)$ is unitary if and only if $\alpha^2+\beta^2+\gamma^2 = 1$. Is $A(\alpha,\beta,\gamma)$ hermitian? Amoung the single qbit gates $X, Y, Z, P(S), T, H$ which of these can be expressed in the above form. Determine the value of α,β,γ for these gates.

Problem 4 Find a pair of 2×2 unitary matrices A and B such that $ABA^\dagger B^\dagger = iX$, where X is the Pauli-X operator.